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New Calculation Method of the Longitudinal Spherical Aberration of Thick Lenses

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ABSTRACT. Using the results of caustic theory, a formula for evaluating the longitudinal spherical aberration produced by a thick lens is proposed. Then, this formula is expressed explicitly in terms of the parameters of the lens by the use of paraxial ray trace technique. Comparison of the corresponding values of the longitudinal spherical aberration, the exact values and the computed values by this formula shows that both values are in good agreement.

Subject term: lens aberrations: ray trace; wavefront curvature; caustic points.

1. Introduction

In the literatures (Chang & Stavroudis 1980, Al-Ahdali 1989, Kassim, Shealy& Burkhard 1989) caustic surfaces, which are associated with an object point, have been considered as the aberrated image of that object point. The motion of the caustic point for axial object was studied while the lens is perturbed by generalized bending(Chang & Stavroudis 1980), and it was found that the magnitude of this motion is proportional to the change in the third order aberration coefficients. Shealy presented a numerical method for evaluating the longitudinal spherical aberration (LSA)(Shealy 1976), which is produced by a lens for the incidence of plane waves by measuring the caustic surface configuration in the sagittal plane. Although the rheory for the evaluation of LSA of thin lenses is based on the third order aberration theory(Born 1980, Jenkins&White 1957, Welford 1986), calculations involving thick lenses, where the lens thickness are not neglected, are often too complicated to be carried out analytically. In lens design, finite ray trace technique, which is based on the use of tedious computational programs, has to be implemented for evaluating the LSA of such lenses. A particular case for evaluating the LSA produced by a spherical refracting surface has been presented by the use of the fifth order approximation(Se-Yuen Mark 1987). Therefore, for the optimization process of a thick lens, an algebraic expression for evaluating the LSA of such lens in terms of its parameters is needed.

In this paper, the results of the caustic theory (Stavroudis 1972)have been implemented to define a function that represents the difference between the principal radii of curvature of the refraeted wavefront from the last surface of the lens and also it represents the differenee between the distance of the caustic points, in sagittal and tangential plane, from this surface along the refracted ray. Then, the relation between the function and the LSA, which is produced by a thick lens, has been investigated for the plane wave incident upon such a lens. To generate the results of this investigation, lenses of different powers have been considered. The evaluation of the principal radii of curvature requires a lot of computational work (Burkhard & Shealy 1981). This paper presents a new algebraic approach to approximate the exact expressions of the principal radii of curvature of the emergent wavefront by the use of paraxial ray trace technique (Kingslake 1978). Then, the exact form of the presented function has heen expressed in an approximated form to ease the computational process of evaluating such a function.

2. Theoretical Analysis

According to the wave aberration theory (Hopkins 1950) the amount of aberration of the emergent wavefront is measured by the deformation of the shape of the wavefront from the spherical shape. This deformation increases with the increase of the ray height at the input plane. Referring to the results of the caustic theory the principle radii of curvature of the emergent wavefront, in sagittal and tangential plane, are equal for the axial ray while the difference between these radii of curvature is observed for the marginal rays. Therefore, for plane waves incident on a thick lens, this study proposed a function Δ, which represents the difference between the principal radii of curvature of the emergent wavefront when it leaves the last surface of the lens (see Fig. 1) and it expressed as

$$\Delta = R_{2s} - R_{2t} \qquad (1)$$

Where R_{2s} and R_{2t} are the principal radii of curvature of the emergent wavefront in this plane. Although the expressions (Burkhard&Shealy 1981), which specify R_{2s} and R_{2t} are given in closed forms, its evaluations requires a lot of computational work. Therefore, in this paper, the paraxial ray trace technique has been employed to express the principal radii of curvature in approximated forms sueh that Eq. 1 can be reformulated in terms of the parameters of the lens to ease the computational process. The mathematical descriptions for expressing R_{2s} and R_{2t} in approximated forms are summarized below.

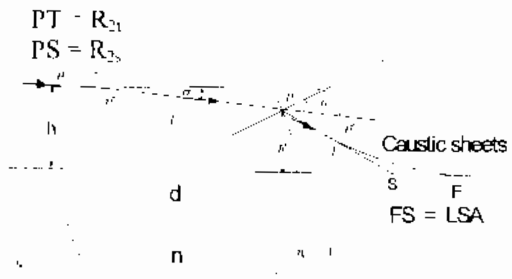

Fig. 1. Ray trace in a thick lens.

2.1. *Refraction at First Surface*

Following the notation given in Ref. (Burkhard&Shealy 1981), the principal radii of curvature of the emergent wavefront from S_1 (see Fig. 1) can he written as

$$R_{1s} = \frac{R_1}{\left(-\gamma_1 \cos\theta_1 + \cos\theta_1^*\right)}, \tag{2}$$

$$R_{1t} = \frac{R_1 \cos^2\theta_1}{\left(-\gamma_1 \cos\theta_1 + \cos\theta_1^*\right)}, \tag{3}$$

Where $\gamma_1 = n_0/n$, θ_1 and θ_1^* are the angle of incidence and refraction of the ray at the first surface, and R_1 is the radius of S_1(see Fig. 1). Using Snell's law, Eqs. 2-3 can be rewritten as

$$R_{1s} = R_1 \sin\theta_1 / \sin(\theta_1 - \theta_1^*). \tag{4}$$

$$R_{1t} = R_1 \sin\theta_1 \cos^2\theta_1^* / \sin(\theta_1 - \theta_1^*). \tag{5}$$

Assuming that the incident ray has a height h above the optical axis (see fig. 1), then Eqs. 4 and 5 can be expressed as

$$R_{1s} = h/\sin\sigma \tag{6}$$

$$R_{1t} = h\cos^2\theta_1^* / \sin\sigma , \tag{7}$$

where $\sigma = \theta_1 - \theta_1^*$.

2.2 *Refraction at Second Surface*

The principal radii of curvature of the emergent wavefront as it leaves the second surface of the lens, R_{2s} and R_{2t}, are expressed as(Burkhard D.G. & Shealy, D. L., 1981)

$$\frac{1}{R_{2s}} = \frac{\gamma_2}{-R_{1p}^*} + \frac{\left(-\gamma_2 \cos\theta_2 + \cos\theta_2^*\right)}{R_2} \tag{8}$$

$$\frac{1}{R_{2t}} = \frac{1}{\cos^2\theta_2^*}\left(\frac{\gamma_2 \cos^2\theta_2^*}{-R_{1p}} + \frac{(-\gamma_2\cos\theta_2 + \cos\theta_2^*)}{R_2}\right) \tag{9}$$

where $\gamma_2 = n/n_0$, θ_2 and θ_2^* are the angle of incidence and refraction at S_2. R_2 is the raduis of curvature of the second surface, $R_{1p} = L - R_{1t}, R_{1s}$, and L is the geometrieal path length of the refracted ray between S_1 and S_2. For a paraxial incidence (Kingslake 1978) at S_1. (see Fig. 1), $L \approx d/\cos\sigma$. Then, using Snell's law for the refracted ray by S_2, Eq. 8 can be written as

$$\frac{1}{R_{2s}} = \frac{\gamma_2}{R_{1s} - (d/\cos\sigma)} - \frac{\sin(\theta_2^* - \theta_2)}{R_2 \sin\theta_2} \tag{10}$$

Since the angle involved in Eq.10 are small, then their sines are nearly equal to the angles themselves in radian (Meyer-Arendt 1984). With this approximation, Snell's law, $n \sin\theta_2^* = n \sin\theta_2$, can be written $n_0\ \theta_2^* = n\ \theta_2$, from which

$$\sin(\theta_2^* - \theta_2)/\sin\theta_2 \approx \gamma_2 - 1 \tag{11}$$

$$\sigma \approx (1 - \gamma_1 h)/R_1 \tag{12}$$

Then combining. Eqs. 6, 11, and 12 with Eq. 10 gives

$$R_{2s} = \frac{n_0\left[1 - p_1 d(1 - h^2 p_2^2 / 6n^2)/n\right]}{P_T + (p_2 p_1^2 d / 6n^3) h^2}, \tag{13}$$

where p_1, and p_2 are the refractive power of the first and the second surface of the lens, which can be, expressed as (Meyer-Arendt 1984)

$$p_1 = n(1 - \gamma_1)/R_1,$$
$$p_2 = n_0 \ (1 - \gamma_2)/R_2,$$

where P_T, the total power of the lens, is given as

$$P_T = p_1 + p_2 - p_1 p_2 d/n,$$

Similarly, the exact expression of R_{2t}, which is given in Eq. 9, can be expressed as follows: Referring to Fig. 1, the angle of incidence, θ_2, of the ray at S_2 can be expressed as

$$\theta_2 = \phi_2 + \sigma, \tag{14}$$

where, for the paraxial incidence on the lens, $\phi_2 \approx h^2 / R_2$, and h^*, the height of the refracted ray when it intercepts S_2, can be approximated as

$$h^* = h - \sigma d. \tag{15}$$

Then, Eq. 14 can be expressed in terms of lens parameters as

$$\theta_2 \approx h \frac{p_2 h^*}{n_0 (1 - \gamma_2)} + \frac{p_1 h}{n}. \tag{16}$$

The angle of refraction at S_2, can be expressed in terms of lens parameters by the use of Snell's law and Eq. 16 as

$$\theta_2^* \approx \sin^{-1}\left[\gamma_2\left(\frac{p_2 h^*}{n_0(1-\gamma_2)} + \frac{p_1 h}{n}\right)\right]. \tag{17}$$

Then, the combining of Eqs. 7,11 with Eq. 9 gives

$$\frac{1}{R_{2t}} = \frac{1}{\cos^2 \theta_2^*}\left(\frac{\gamma_2 \sin\sigma \cos^2 \theta_2}{h\cos^2 \theta_1^* - d\tan\sigma} + \frac{p_2}{n}\right), \tag{18}$$

where $\theta_1^* \approx \sin^{-1}[(\gamma_1 p_1 h)/(n - nL_0)]$. Then, Eq. 18 can be rearranged in terms of the lens parameters as

$$R_{2t} = \frac{n_0 C_2 (1 - \gamma_2^2 C_1^2)}{p_2 C_2 + p_1 h (1 - C_1^2)}, \tag{19}$$

where

$$C_1 = \frac{p_1 h}{n} + \frac{p_2 h^*}{n_0(1-\gamma_2)},$$

$$C_2 = h(1 - \frac{p_1 d(1 - p_1^2 h^2 / 6n^2)}{n} - (\frac{\gamma_1 p_1 h}{n - n_0})^2).$$

Finally, combining the Eqs. 13,19 with Eq.1 gives

$$\Delta = \frac{n_0(1 - p_1 d(1 - h^2 p_1^2)/n)}{P_T + p_2 p_1^3 h^2 d/(6n^3)} - \frac{n_0 C_2(1 - \gamma_2^2 C_1^2)}{p_1(1 - C_1^2)h + p_2 C_2}. \tag{20}$$

Eq. 20 represents an approximated expression for the difference between the principal radii of curvature of the emergent wavefront from a thick lens when plane waves incident upon it.

3. Results

In this study, plane waves incident on a lens, surrounded by air, which has the following parameters: surfaces curvature 0.0715 and -0.0564 cm^{-1}, thickness 3.0 cm, refractive index 1.5, and an effective focal length 15.0 em. Then, finite ray trace technique[15] has been used to compute the exact values of the LSA of this lens, where the values of Δ are computed by the use of Eq. 1 (see Table1.). comparison of the corresponding value, for the same height of the incident ray at the input plane which is located at the vcrtex of the first surface of the lens, shows that the value of LSA almost equals half of the value of Δ over the range of ray height 0-2.5 cm. Similar study has been carricd out to investigate the relation between the LSA and Δ for various lenses of different effective foeal length and of the same thickness (see Table 2.). For the ray height 2.5 cm, calculation shows that the ratio of LSA/Δ almost equals 0.5 for all considercd lenses. As a result of this study, one ean evaluate the LSA of a thick lens by the evaluation of half of Δ without the loss of accuracy.

Table 1: The exact values of L.S.A and Δ as function of the ray height at the input plane

Ray height(cm)	LSA(cm)	Δ (cm)	LSA/Δ
0.25	0.0059	0.0118	0.5000
0.50	0.0237	0.0474	0.5000
0.75	0.0534	0.1068	0.5000
1.00	0.0951	0.1890	0.5016
1.25	0.1490	0.2970	0.5017
1.50	0.2154	0.4276	0.5037
1.75	0.2945	0.5830	0.5051
2.00	0.3866	0.7630	0.5067
2.25	0.4922	0.9678	0.5086
2.50	0.6118	1.1980	0.5107

Table 2: The exaet valucs of L.S.A and Δ as a function of the effective focal length of a thick lens.

Focal Length(cm)	L.S.A(cm)	Δ (em)	L.S.A/Δ
20.0	0.3130	0.6158	0.5083
25.0	0.2555	0.5056	0.5053
30.0	0.2160	0.4286	0.5040
35.0	0.1871	0.3720	0.5030
40.0	0.1651	0.3286	0.5024
45,0	0.1476	0.2942	0.5017
50.0	0.1336	0.2664	0.5015

To investigate the validity of Eq. 20 for evaluating the LSA of a thick lens, a comparison between the exact values of the LSA, produced by the previous lens, and the approximated values of $\Delta/2$, computed by Eq. 20, has been carried out for the range of the ray height 0-2.5 cm (see Fig. 2). Results show that both values are in good agreement over the range of the height of the incident ray at the inout plane. Also, for a ray height 2.5 cm, a comparison between the corresponding values of the LSA and $\Delta/2$ as a function of the lens power (see Fig. 3) shows that both values are in good agreement.

Referring to the previous results, Eq. 20 may be used for evaluating the LSA of a thick lens without using the complicated finite ray trace technique.

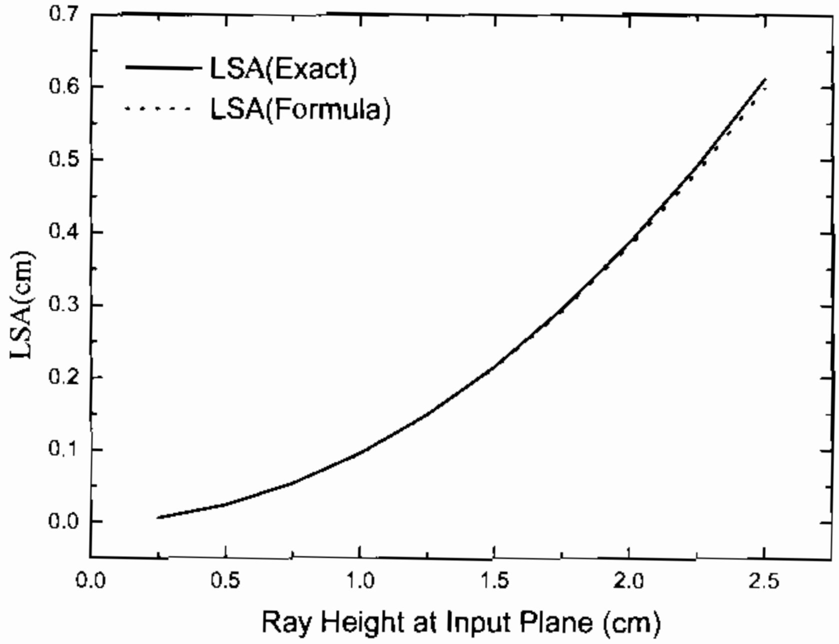

Fig. 2. Comparison of the exact values of L.S.A and the approximated values of $\Delta/2$ as a function of the ray height at the input plane.

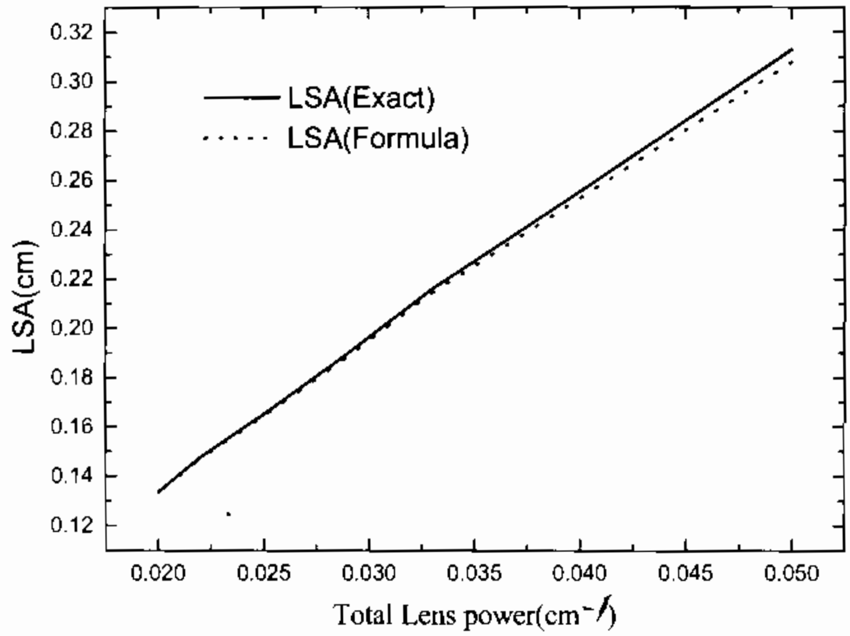

Fig. 3. Comparison of the exact values of L.S.A and the approximated values of $\Delta/2$ as a function of the lens power.

4. Conclusion

This study has related the longitudinal spherical aberration, which is produced by a thick lens for a plane waves incident upon it, with the difference between principal radii of curvature of the emergent wavefront from the last surface of the lens.The study shows that the algebraic expression of this difference can be used to evaluate the LSA of a thick lens. Compared to other numerical techniques, which are used in lens design, the approximated formula for this difference may ease the evaluation of the LSA of such lens.

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طريقة جديدة لحساب ظاهرة الزيغ الكروي الطولي للعدسات السميكة

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المستخلص. باستخدام نظرية المحرق يمكن إيجاد صيغة الزيغ الكروي الطولي الناتج من العدسة السميكة .

وباستخدام هذه الصيغة التي يمكن التعبير عنها بدلالة معاملات العدسة، وباستعمال تقنية تتبع أثر الشعاع المحوري.

وبمقارنة القيم المتناظرة للزيغ الكروي الطولي فيما بين القيم الحقيقية و القيم المحسوبة بهذه الصيغة اتضح أن هذه القيم المقارنة في توافق تام.